Simplified Solar Heating Model

For

SolarAttic™ PCS1 Pool Heater

Prepared from notes of the "1994 Scoping Analysis" as performed by Professor Ephraim M. Sparrow, University of Minnesota, Minneapolis, MN.

Heat Supply To Attic Air:

1. Direct solar.

Use 250 Btu/hr ft^2 incident normal to the sun's rays. The angle of incidence varies throughout a given day and also depends on latitude and on the season. Use a factor of 0.7 to account for the non-normal incidence. As a best case, assume that all the incident solar is absorbed (i.e., $d_S = 1$). If a 1500 ft^2 roof is everywhere incident by direct solar, then

$$250 \times 0.7 \times 1500 = 262{,}500 \text{ Btu/hr.}$$

This would be valid for an unshaded, low-pitch roof. Perhaps a more typical roof would be half shadowed. For such a roof, the absorbed solar would be $\cong$ 130,000 Btu/hr.

2. Scattered Solar.

Use 30 Btu/hr - ft^2 as a typical value. For total absorption on a 1500 ft^2 roof

$$30 \times 1500 = 45{,}000 \text{ Btu/hr}$$

3. Total Range of solar absorpton.

175,000 - 307,500 Btu/hr

Heat Losses From Attic Roof:

1. Convection losses.

Assume roof outside surface temperature is 115^{o} F and that the air ambient is 85^{o} F. Convective heat loss equation:

$$Q_{conv} = \quad \Delta T/R, \qquad R= 1/hA$$

where R is the thermal resistance without wind,

$h \cong 0.7$ Btu/hr-ft2 -^{o}F. So that

$$Q_{\ conv} = \frac{\underline{(115 \quad - 85)}}{\frac{1}{(0.7)(1500)}} = 31{,}500 \text{ Btu/hr}$$

For moderate wind, use $h \cong 2$ Btu/hr - ft^{2} - ^{o}F

Then,

$$Q_{loss} = \frac{\underline{(115 - 85)}}{\frac{1}{(0.7)(1500)}}$$

$$Q_{loss} = 90{,}000 \text{ Btu/hr}$$

2. Radiation losses.

If the diffuse sky radiation is characterized by T_{sky}, then,

$$Q_{\ loss} = E\sigma A\ (T^{4}-T_{sky}{}^{4})$$

This equation is for infrared radiation. The emissivity E is about 0.85 for both black and white roof surfaces; $\sigma = 0.1712 \times 10^{-8}$ Btu/ hr ft^{2} ^{o}R^{4}. Both T and T_{sky} are in degrees Rankine, $^{o}R = {}^{o}F + 460$. Then,

$$Q_{loss} = (0.85)(0.1712 \times 10^{-8})\ (1500)(575^{4} - 460^{4})$$

Q_{loss} = 141,000 Btu/hr.

Heat Balance:

Max Q_{loss} = 90,000 + 141,000
= 231,000 Btu/hr

Min Q_{loss} = 31,500 + 141,000
= 172,500 Btu/hr

Range of losses: 172,500 - 231,000 Btu/hr

Range of solar absorbed: ($\alpha_s = 1$)

175,000 - 307,500

Conclusion: There are clearly many cases where the PCS1 system will work (i.e. provide = or > 90,000 Btu/hr for heating a pool.)....

Comment: On an outside sunny day with 85 °F temperatures, the roof's surface temperature can easily reach 130 - 150 °F [At least on my own brown asphalt roof in Elk River, MN (EGP)]. However, the PCS1 will function as an internal heat sink drawing solar energy inwardly. Also, thermal air streams on the roof's surface could easily be drawn inwardly through the passive vents as heat energy travels from hot to cold. The interior of the attic is no longer building resistance to thermal conductivity as it would be without the PCS1 working to lower its internal temperature.

$$\mathbf{Qconv} := \frac{115 - 85}{\left(\frac{1}{2 \cdot 1947}\right)}$$

$$\mathbf{Qconv} = 116820 \ \blacksquare$$ **BTUs/Hour** moderate wind

C.2. Radiation Losses

$$\mathbf{QLoss} := (.85)\cdot(.1712\cdot 10^{-8})\cdot(1947)\cdot(575^{4} - 460^{4})$$

$$\mathbf{QLoss} = 182855 \ \blacksquare$$ **BTUs per Hour**

D. Heat Balance

Max Q_{Loss} = 116,820 + 182,855 = 299,675 BTUs/Hr

Min Q_{Loss} = 40,887 + 182,855 = 223,742 BTUs/Hr

Range of Losses : 223,742 - 299,675 BTUs/Hr

Range of Solar Absorbed: 228,772 - 399,135 BTUs/Hour

VI. Utility Co. Best Case Scenario:

Max Absorption & Minimum Losses

Maximum absorption = 399,135
Minimum loss = 223,742

399,135 - 223,742 = 175,393 BTUs per Hour

Available to PCS1

VII. Utility Co. Worse Case Scenario:

Min Absorption & Maximum Losses

Minimum absorption = 228,772
Maximum loss = 299,675

Losses equal or exceed gain (Utility Co. Model)

BUT, this obviously doesn't apply to hot attics!
AND, the PCS1 install criteria calls for a hot attic.

I.E. Attic Temp ≥ 100° F on 70° F Sunny Days.
See #23 pg. 8 of Common Questions in Ref. section of Technical Manual.
See specifications sheet for PCS1.

VIII. Utility Co. Pool Requirements

Degrees	Btus Req.
1	6750
2	13500
3	20250
4	27000
5	33750
6	40500
7	47250
8	54000
9	60750
10	67500

IX. Conclusion

The largest known daily temperature increase in current PCS1 installations is 8° F in a single day. This is the Orange Park, Florida installation. This pool owner's data would suggest, that, under his particular circumstances, the PCS1 is capable of operating around its sensible heat nominal design of 60,000 Btus per hour. Note: This is a large 20,000 gallon pool.

This scoping analysis model indicates that the PCS1 could easily raise the pool from 8 - 10° F under optimum solar conditions. This is in direct conflict with the Utility Company's math model that suggests the PCS1 could not raise the pool 1.8° F. The main problem I see with this traditional model (scoping analysis) is the QLoss calculations which somehow need to reflect the presence of a much cooler attic with the PCS1 in operation.

This simplified solar math model demonstrates that 175,393 Btus per hour could be available for a system that only seeks 20,000 - 60,000 Btus per hour to satisfy pool owners.